

Economics 765

Spring 2025

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Course Outline

The course is directed to students who wish to learn the mathematical techniques used in modern finance theory. The course will also include the basic theory of asset pricing, in particular, the pricing of derivative assets, such as options. If time permits, more elaborate models will also be discussed. The introductory material starts with measure theory, a topic not always treated in courses of mathematics for economists. Measure theory is however a necessary prerequisite for the sort of probability theory needed for financial applications. In particular, we will treat stochastic processes in continuous time, of which the simplest example is Brownian motion.

A brief list of the foundational topics we will treat is as follows.

- Measure theory and the Lebesgue integral;
- Probability based on Sigma-algebras and filtrations;
- Conditional expectations;
- Theory of martingales and arbitrage-free pricing;
- Markov processes and stopping times;
- Generalised probability density and the Radon-Nikodym theorem;
- Brownian motion and Ito's stochastic calculus;
- Stochastic differential equations;
- Kolmogorov's backward and forward equations;
- Girsanov's theorem.

On the more applied side, we will consider

- Hedging a portfolio;
- European and American options;
- Arbitrage-free pricing;
- Specific models, such as Black-Scholes, Cox-Ingersoll-Ross.

Textbook

We will follow the two-volume set entitled *Stochastic Calculus for Finance*, by Steven Shreve, in the Springer Finance series. The first volume contains no sophisticated mathematics, but allows readers to develop valuable intuition by a detailed treatment of the so-called binomial model, the simplest of all models of derivative pricing. We will make use of many of the examples in that volume. The second volume is where most of the material for the course is to be found. It combines mathematical developments with some quite sophisticated financial models.

Academic Honesty

You'll have seen the following in all of your course outlines, because the McGill Senate requires that it should appear in all of them. I used to think of it as a pure formality, but a disturbing number of cases of plagiarism have been detected in recent years, not especially at McGill, but in other North American universities. So, please take seriously all the admonitions in the following text.

- 1) Right to submit in English or French written work that is to be graded [approved by Senate on 21 January 2009]: In accord with McGill University's Charter of Students' Rights, students in this course have the right to submit in English or in French any written work that is to be graded. This right applies to all written work that is to be graded, from one-word answers to dissertations.
- 2) According to Senate regulations, instructors are not permitted to make special arrangements for final exams. Please consult the calendar, section 4.7.2.1, General University Information and Regulations, at <http://www.mcgill.ca>.
- 3) Academic Integrity statement [approved by Senate on 29 January 2003]: McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures.

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Et en français:

L'université McGill attache une haute importance à l'honnêteté académique. Il incombe par conséquent à tous les étudiants de comprendre ce que l'on entend par tricherie, plagiat et autres infractions académiques, ainsi que les conséquences que peuvent avoir de telles actions, selon le Code de conduite de l'étudiant et des procédures disciplinaires (pour de plus amples renseignements, veuillez consulter le site

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